Networks of Decision-Making and Communicating Agents: A New Methodology for Design and Evaluation of Organizational Strategies and Heterarchical Structures*

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ABSTRACT

In this paper, we explore the decomposition of organizational processes and decision-making, the coexistence of organization's command and communication networks, and their effects on team performance. We present a methodology to design mission-based strategies and novel heterarchical organizational structures based on exploring information/command transfer and processing in the organizations. This methodology allows synthesising alternative C3I organizational structures, which outperform traditional hierarchies in environments with heavy information volume, scarce resources, and strict knowledge & communication constraints.

1. Introduction

As a response to volatile environments, organizations struggle to balance stability against flexibility, specialization against generalization, and centralization against decentralization (Alstyne, 1997). A traditional hierarchy has a topology that largely restricts interactions among members of the organization to direct superior/subordinate interactions and whose number of levels is determined by the limits of span of command (Alberts, 2003). Its approach to command and control is characterized by centralized planning, decomposition of tasks, and control processes that largely rely on deconfliction. Hierarchies spaw*n stovepipes, which are vertical, tightly coupled component organizations that are optimized for a narrowly focused objective. The systems that support hierarchies are built and controlled by stovepipes, making interoperability difficult to achieve. Furthermore, information flows in hierarchies mirror the hierarchical structure and are largely confined to stovepipes that originated or collected the information of interest.

A heterarchy is an emergent, self-organizing form that resembles a network or a fishnet. It has lateral or distributed authority, has no fixed superior DM and has bi-directional relationships among DMs. A Decision-maker may become a superior based on his/her abilities and nature of the mission. Heterarchies involve relationships of *interdependence*.

An organization, which utilizes the beneficial characteristics of both hierarchy and heterarchy and can evolve over time, is termed a *hybrid* organization. Hybrid networked organizations encourage appropriate interactions between and among any and all members. Its approach to command and control breaks the traditional C2 mold by uncoupling command from control. Command is involved in setting the initial conditions and providing the overall intent. Control is not a function of command, but an emergent property that is a function of the initial conditions, the environment, and the adversaries. Such organizations have attributes to be agile. This is because the agility requires that the available information is combined in new ways, that a variety of perspectives are brought to bear, and that assets can be employed differently to meet the needs of a variety of situations. Hybrid organizations are particularly well-suited to deal with uncertainties because they make more of their relevant knowledge, experience, and expertise available.

In this paper we explore the decomposition of organizational processes and decision-making, the coexistence of organization's command and communication networks, and their effects on team performance. We investigate the interrelationships between observation, command, communication, and task

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execution in the organization, and describe a design methodology for constructing optimal strategy for information/command processing and communication using minimum flow cost problem formulation (static case) and information-constrained reward projection (dynamic case) modeling approaches. Proposed methodology presents a tool for the design and comparison of hierarchical, heterarchical and hybrid organizational structures, sensitivity analysis, and evaluation of design parameter tradeoffs.

The paper is organized as follows. In Section 2, we provide motivation behind employing information-command model. In Section 3, a simplified modeling based on single-network formulation of information processing problem is presented. In this Section, we outline the approaches for static and dynamic strategy-structure modeling (solution approaches presented in the Appendix). A 2-network (coupled information and command) problem formulation is presented in Section 4. An example of application for our design methodology is described in Section 5, with example of sensitivity analysis presented in Section 6. The paper concludes with summary and future extensions in Section 7.

2. Motivation

2.1. Example: event-driven missions

Let's consider the organization designed to execute a mission in a highly uncertain environment. One of the approaches is to identify events pertinent to changes in the environment, and specify the tasks that must be executed by the organization in order to respond to those events. An example of event in a military context is the appearance of an unidentified object on the radar, in which case a response to this event is the allocation of assets to identify the target (neutral or hostile), and assignment of assets to prosecute and/or engage it (in case it is hostile). An example of an event in a business context is an increase in the sales of a product, which requires an organization to perform the following tasks: identify the causes of the increase (increase in the quality of a product, decrease in competition on the market, increase in the number of consumers, etc.), check inventory levels, increase the production, etc. In those situations, the organization must observe the events first (*monitor* environment), decide what tasks to perform (*information processing*, or *command generation*), and then distribute tasks to organizational elements for processing (*command processing*). The information (from observations) and command (from decision-making) is propagated in the organization via communication between organizational nodes (agents). On one hand, we can assume that the command is generated only when the environment events are observed by the organizational nodes and the corresponding information enters the organization. On the other hand, we can assume that communication (interactions among agents) occurs only as a result of observation and/or information processing (command generation). Hence, the organizational operation is decomposed into mission monitoring, information processing (command), and command processing (task/command execution).

2.2. Interactions in the organization

A. Interaction triggers

As noted above, the triggers of information transfer include the observations by organizational nodes of the environmental events, and the information generated by organizational elements in their operation during mission execution. The triggers of command transfer may include the receipt of information. The latter is converted into the command and transferred to the destination nodes for execution.

B. Communication types: command and information

In this paper, we consider the information and command transfers, which comprise correspondingly the information transfer network and command transfer network. The information received by organizational observers must be transferred to and then processed (converted into command) by organizational nodes (decision-makers). This command must theretofore be transferred to processing elements of the organization for execution. As a result, the efficiency of information-to-command transfer is tightly coupled with efficiency of command execution. Therefore, the command and information networks must be modeled as a coupled system representing a single organizational network.

In this paper, we first describe a simplified problem of finding an optimized strategy for *information network*, which deals with information transfer and processing only (Section 3). This problem is approached using two related modeling techniques: static model (Subsection 3.5), and dynamic model (Subsection 3.6). The solution approaches developed for these problems can be applied to the coupled information/command networks modeling (Section 4), since the difference between 2-network problem and single network problem is only in the structure of the network nodes and values of cost and constraint parameters.

3. Information network problem

3.1. Organizational structures: definitions

In this subsection, we describe main concepts of the problem of modeling the organizational structure, which

include (i) events, (ii) information flow, (iii) organizational agents, and (iv) agent network.

A. Events

A mission is defined as a collection of N events $\{E_1,...,E_N\}$. For each event E_n, an event information volume e_n is defined. An event can be viewed as a set of tasks that the decision-makers must accomplish. Therefore, information flow received from the event must be propagated through the agent network and processed (consumed) by agent nodes. An example of information processing is an execution of tasks included in the event. To simplify our results, we assume that incoming events are of the same type.

B. Information flow

The volume of information from events is transferred through the network and consumed by agents. The information is assumed to be separable, and can be routed via different paths in the network (see Section 3.5.E on non-separable information). This defines an *information flow* in the organization.

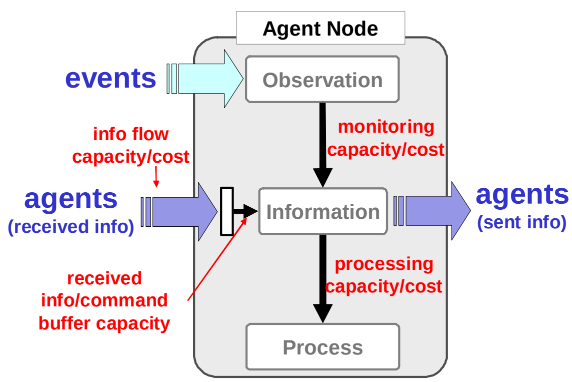

Figure 3.1. Agent-flow model

C. Organizational Agents

An organization is represented as a network of M nodes, termed *agents*, with decision-making capabilities. Agents accomplish the following tasks:

- *monitor* the events, and receive their information volume;
- *receive* the flow/information from other agents;
- *fuse* the received flows;
- conduct *decision-making* to separate the received information into the flow consumed/processed by the agent and the flow to be sent to other agents; this can be considered as *flow decomposition task*;
- *process* the flow that is consumed by them;
- *send* the other flow to the rest of the team.

According to the above, a model of the flow in the agent network is outlined in Fig. 3.1.

Therefore, the following parameters are defined for each agent A_i:

- *Monitoring capacity* m_i^{MAX}; an agent can monitor and receive information from events of up to m_i^{MAX} units;
- *Processing capacity* p_i^{MAX}; an agent can process of up to p_i^{MAX} units of information;
- *Information fusion capacity* r_i^{MAX}; an agent can receive and must fuse total up to r_i^{MAX} units of information; this parameter can also be referred to as *fusion efficiency*;
- *Processing efficiency* – the gain obtained by processing a unit of information at the agent: g_i;
- *Event-to-agent assignment*: $u_{n,i} \in \{0,1\}$, $\sum_{i=1}^{M} u_{n,i} = 1$; these parameters define the observers or sources of the flow in the organization.

The parameters above are generically described, and their specific treatment depends on the modeling methodology employed for the design problem. We define unique capacities for monitored information and received information since agents have limited capabilities and must allocate them to monitoring and decision-making. Due to limited workload capability, the sum of capacities of monitored and processed information would be limited. If agents' observation capacities were unlimited, the information would have to be received directly by agents with highest decision-making efficiency to maximize processing gain. Those agents will process all received information, and information flow in the network would not occur.

D. Organizational Network

Agents receive and send information in the network, which is a collection of links/channels that connect the

organizational agents and allow them to exchange the information. Therefore, we can define an agent network in terms of link capacities $c_{i,j}$, with $\sum_{i,j=1}^{M} c_{i,j} \le C$. Link capacities $c_{i,j}$ determine the maximal volume of information that can be transferred from agent A_i to agent A_j.

3.2. Problem constraints and objectives

Workload constraints: The processing and monitoring capacity defined above usually refer to the trade-off that an agent exercises in dedicating its time to internal information (processing capacity) and external information (monitoring capacity). Therefore, $m_i^{MAX} + p_i^{MAX} = \Omega$, where Ω identifies agent's total workload capacity (we assume for simplicity identical agents).

Monitoring constraints: Defining the flow received by an agent A_i from events as $m_i; i = 1,...,M$, we have that $m_i \le m_i^{MAX}$.

Fusion constraints: Defining the flow received by an agent A_i as $r_i; i = 1,...,M$, we have that $r_i \le r_i^{MAX}$.

Processing constraints: Defining the flow processed by agent A_i as $p_i; i = 1,...,M$, we have that $p_i \le p_i^{MAX}$.

Link flow constraints: Defining the flow along the link from A_i to A_j as $x_{i,j}; i, j = 1,...,M$, we have that $x_{i,j} \le c_{i,j}$.

Link capacity constraints: The agents in the organization have limited communication capabilities. Hence, there might be additional constraints imposed on the flow in the agent network. Defining the maximum and minimum capacity along the link from A_i to A_j as $c_{i,j}^{MAX}, c_{i,j}^{MIN}; i, j = 1,...,M$, we have: $c_{i,j}^{MIN} \le c_{i,j} \le c_{i,j}^{MAX}$.

Objectives: The network design problem is to determine the link capacities in the information network, and is coupled with strategy design problem that must find the optimal routing and processing assignment for the information. To identify the cost of the network, we define $\alpha_{i,j}$ - the cost per unit of capacity on the link, with the total cost equal to $\sum_{i,j=1}^{M} \alpha_{i,j} c_{i,j}$. Various objective functions might be considered: minimization of total network cost, maximization of reward from information processing, minimization of information delay, etc. These problems can be addressed by various modeling techniques described in next sections.

3.3. Defining agent parameters

The parameters included in the description of agents must be carefully defined. While capacity constraints on the links among agents are due to physical limitations of the network and individual interaction principles, the origin of link costs and agent processing gains is not immediately clear. In this subsection, we describe an analytic approach for computing those parameters. Note that agent processing gain is different for information- and command-type flow problems.

We define the limited knowledge of agent A_i in terms of its *expertise vector* $[\omega_1^i, ..., \omega_\Phi^i]$, where ω_k^i is an expertise in processing of information type k, and Φ is the number of defined information types. Similar to (Ferreira and Sah, 2001), this models the width and depth of agents' expertise. The more *generalized* A_i is (it can handle information in a larger scope), the more non-zero entries ω_k^i are in its expertise vector. The more *specialized* A_i is for a specific information type k (it can handle information in higher depth), the larger ω_k^i is. Due to limited capability, the agent exercises a trade-off between generalization and specialization, which we define using expertise constraint $\sum_{k=1}^{\Phi} \omega_k^i \le \Delta$, where Δ is a bounded talent of the agent.

A. Gain in information flow networks

This type of the network describes the transfer of event information to the nodes that convert it to command. Therefore, the gain from processing the information and converting it into command relates to the efficiency of the agent to handle the information in a large scope. In this paper, we define the agent gain for information processing equal to the measure of variability of agent expertise termed *knowledge width* (generalization capabilities). We use a variant of normalized entropy, which is scaled with mean expertise:

$$g_i = \frac{1}{\log \Phi} \left[\sum_{k=1}^{\Phi} \omega_k^i \log \sum_{k=1}^{\Phi} \omega_k^i - \sum_{k=1}^{\Phi} \omega_k^i \log \omega_k^i \right].$$

Here, $g_i \in [0, \Delta]$, with maximum reached for equally distributed expertise: $\omega_k^i = \frac{\Delta}{\Phi}; \forall k = 1,...,\Phi$. This is

equivalent to the maximally generalized agents. The gain is minimal for maximally specialized agents (that is, when $\omega_j^i = \Delta, \omega_k^i = 0; k \neq j$).

B. Information transfer cost

During information transfer, both loss in timeliness and precision occur. The loss of precision might happen when agents with different knowledge communicate, and this loss increases with increase in the knowledge gap between communicating agents. Therefore, we relate the cost of information transfer to the loss of information precision. When type k information is transmitted from agent A_i to A_j, there is no information loss if $\omega_k^j \geq \omega_k^i$. In this case, the receiving agent A_j is more specialized in understanding type k information, and therefore has no problem understanding the meaning of the message conveyed by A_i. However, when $\omega_k^j < \omega_k^i$, there is a loss in precision since the sending agent A_i is more specialized, and receiving agent A_j may lose the specific details contained in the message. The loss increases proportionally to the gap in the knowledge, i.e. $\omega_k^i - \omega_k^j$. We can write $\alpha_{i,j} = \sum_{k=1}^{\Phi} \mathsf{F}(\omega_k^i - \omega_k^j)$, where function $\mathsf{F}(\cdot)$ is monotonically increasing and $\mathsf{F}(x) \geq 0, x > 0$ and $\mathsf{F}(x) = 0, x \leq 0$. For the same talent, the cost $\alpha_{i,j}$ is maximal between agents fully specialized for different information types.

3.4. Two models of information transfer/processing

In this subsection, we discuss the two methodologies used to model the way an information is processed and communicated in the organization: (i) static model; and (ii) system dynamics model.

A. Static model

The static model assumes the instantaneous information transfer based only on the flow constraints and cost definitions.

B. System dynamics model

The system dynamics model represents the information transfer as an iterative process, accounting for time dimension in the reward definition, incorporating local agents' decision rules and limited information, and considering the problem constraints per single iteration step.

3.5. Network model based on static problem formulation

A. Problem formulation

The difference of static problem formulation from dynamic one is in the treatment of the information flow in the network and modeling of the agent decision-making. In a static network model, the information is regarded as a static flow in the network, with edge capacity constraints indicating the maximal total amount of the information that can be passed through a link (equivalent to agents one-to-one communication constraints), and agent node capacity constraint indicating the maximal flow throughput in the node. We can also model the loss of the flow on the links.

B. Constraints

Additional constraints must be considered to properly define the problem. First, we notice that in the static problem formulation, the capacities $c_{i,j}$ play only restrictive role, and can be removed from consideration. When the flow through the network is determined, the capacities equal to the total link flow are assigned to the links. As a result, the network cost is equal to $\sum_{i,j=1}^{M} \alpha_{i,j} x_{i,j}$, and we have the flow constraints: $c_{i,j}^{MIN} \leq x_{i,j} \leq c_{i,j}^{MAX}$. Second, we notice that flow conservation constraints must be satisfied. Therefore, $\sum_{n=1}^{N} e_n u_{n,i} + \sum_{j=1}^{M} x_{j,i} = \sum_{j=1}^{M} x_{i,j} + p_i$.

Third, since monitored flow is equal to $m_i = \sum_{n=1}^{N} e_n u_{n,i}$, we have the following event monitoring constraints: $\sum_{n=1}^{N} e_n u_{n,i} \leq m_i^{MAX}$.

Fourth, given the fact that received flow from other agents is equal to $r_i = \sum_{j=1}^{M} x_{j,i}$, we have the following throughput constraints: $\sum_{j=1}^{M} x_{j,i} \leq r_i^{MAX}$

C. Objectives

In this problem formulation, we need to design flow routing rules (a *strategy*) and the supporting network to maximize the total reward. The latter is calculated as the gain obtained by processing the information by agent nodes minus the loss equal to the cost of the agent network:

$$reward = \underbrace{\sum_{i=1}^{M} g_i p_i}_{gain} - \underbrace{\sum_{i,j=1}^{M} \alpha_{i,j} x_{i,j}}_{loss}$$

The optimization is subject to constraints described above, and is over event allocation variables $u_{n,i} \in \{0,1\}$, agent processing variables $p_i \geq 0$, and link flows $x_{i,j} \geq 0$.

D. Problem formulation

The problem of optimal information strategy design is formulated in terms of both the unknown event-to-agent assignment $u_{n,i}$ and link flows $\{x_{k,m}\}$ (which are determined by the flow routing) to maximize the network reward $\sum_{i=1}^{M} g_i p_i - \sum_{i,j=1}^{M} \alpha_{i,j} x_{i,j}$ subject to individual agents' processing, fusion, and flow constraints. The problem becomes:

$$\max \sum_{i=1}^{M} g_i p_i - \sum_{i,j=1}^{M} \alpha_{i,j} x_{i,j}$$

$$s.t. \begin{cases} \sum_{n=1}^{N} e_n u_{n,i} + \sum_{j=1}^{M} x_{j,i} = \sum_{j=1}^{M} x_{i,j} + p_i \\ c_{i,j}^{MIN} \leq x_{i,j} \leq c_{i,j}^{MAX} \\ \sum_{j=1}^{M} x_{j,i} \leq r_i^{MAX} \\ \sum_{n=1}^{N} e_n u_{n,i} \leq m_i^{MAX} \\ \sum_{i=1}^{M} u_{n,i} = 1 \\ u_{n,i} \in \{0,1\}, x_{i,j} \geq 0, p_i \geq 0 \end{cases}$$

E. Modeling issues: simplifying problem formulation

Our strategy design problem can be reformulated as minimum cost flow problem (MCFP), in which we redefine the network structure introducing some artificial nodes to redefine node throughput constraints, information processing and gain constraints as link capacity and cost among the nodes in this modified network.

First, the throughput constraints on the nodes can be modeled by introducing additional nodes and entering the corresponding constraints at the added link (see Fig. 3.2). Second, we introduce additional source nodes $E_1,...,E_N$ for each event (with supply equal to the volume of event's information e_n), and connect them to all agent nodes, which in turn are connected to the added artificial "process" node with the demand equal to $\sum_{n=1}^{N} e_n$ (the agent nodes have therefore 0 demand). The flow from an agent to the "process" node is equivalent to processing of information at the agent node. Hence, the link from an agent to "process" node is assigned capacity equal to agent's processing capacity constraints and link cost equal to $-g_i$ (negative of reward of processing a unit of information).

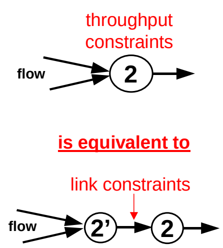

Figure 3.2. Modeling node throughput flow constraints

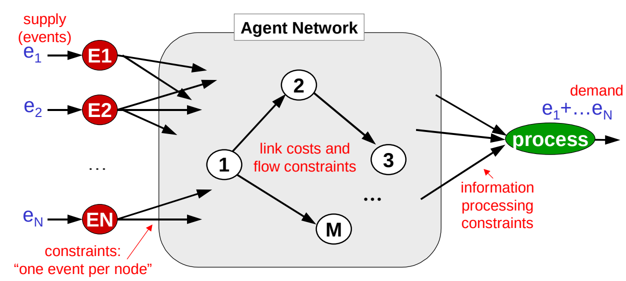

Figure 3.3. Static network model (no flow loss)

The information sources (event nodes) cannot split their information, and must send it to only a single agent node. The flow from event node to the agent node is then equivalent to assigning this event to an agent.

Therefore, without loss of generality, we assume from now on that we deal with a problem of information flow with link capacity, cost, supply, and demand constraints. We skip agent throughput constraints (such as capacity of received information) for simplification of notations. The event assignment constraints introduce additional restrictions on node supply.

F. Solution approaches

Hence, the problem can be reformulated as min flow cost problem in the network of $N+1$ nodes with event allocation, link constraints and fixed demands (the only node with demand will be the "process" node). The problem can generically be described as follows:

$$\min \sum_{i,j=1}^{M+1} \alpha_{i,j} x_{i,j}$$

$$s.t. \begin{cases} s_i + \sum_{j=1}^{M+1} x_{j,i} = \sum_{j=1}^{M+1} x_{i,j} + d_i \\ c_{i,j}^{MIN} \le x_{i,j} \le c_{i,j}^{MAX} \\ \sum_{n=1}^{N} e_n u_{n,i} = s_i \\ s_i \le m_i^{MAX} \\ \sum_{i=1}^{M+1} u_{n,i} = 1 \\ u_{n,i} \in \{0,1\} \end{cases},$$

where s_i, d_i are supply and demand at nodes.

Solution to the above problem requires either removing the integrality constraints for $u_{n,i}$ variables, or separating the flow cost problem and the allocation constraints $\sum_{n=1}^{N} e_n u_{n,i} = s_i, \sum_{i=1}^{M} u_{n,i} = 1$. For details, see Appendix A.

G. Non-separable information

In our problem formulation, we assumed that the information routed through the network can be modeled as a separable flow (it can be split via different paths). This might not be the case with certain flow types, where split information is meaningless and might introduce confusion. In such settings, the solution is sought to find an unsplittable flow from event source nodes to a "process" node for a network in Fig. 3.3. By "unsplittable" we mean that only a single path from a source node to a destination node is used to transfer/rout the flow. A similar problem formulation applies to events that consist of several information packets that cannot be split. The problem is equivalent to a single source/multiple destination unsplittable flow problem, and is NP-hard. Therefore, polynomial approximation algorithms must be used in order to find the solution (Azar and Regev, 2001). Note, that the settings provide the solution to both event-to-agent allocation and flow routing problems.

3.6. Network model based on system dynamics problem formulation

A. Problem formulation

The static problem formulation does not account for the time factor in the transfer of flow in agent network. We can reformulate the problem by considering a single-step flow transition, and introduce the time decay factor in the objective of gain maximization. The difference of system dynamics problem formulation is also in the approach to flow routing. Here, we can explore locally optimal decision-making depending on the dynamic state of the system. The formulation considers the dynamic state of parameters at time $[k]$. All flow-limiting constraints refer to the single time step. To simplify the problem, we assume that event-to-agent allocation is known, and satisfies the "monitoring" constraints.

The following parameters are considered:

- $r_i[k]$ - received flow at agent A_i at time $[k]$;
- $p_i[k]$ - processed flow at agent A_i at time $[k]$;
- $\pi_i[k] \in [0,1]$ - processed flow rate at agent A_i at time $[k]$ (decision variable; it is equal to the portion of information from received buffer that is processed by the agent at the next step);
- $x_{i,j}[k]$ - sent flow from agent A_i to agent A_j at time $[k]$;
- $\pi_{i,j}[k] \in [0,1]$ - sent flow rate from agent A_i to agent A_j at time $[k]$ (decision variable; it is equal to the portion of information from received buffer that is sent to agent A_j at the next step).

This problem formulation also allows modeling the flow loss on the links in agent network by defining $l_{i,j} \in [0,1]$ - amount of flow loss per unit of information transferred on the link from agent A_i to agent A_j. Therefore, viewing $\alpha_{i,j}$ as the cost of unit of lost information, we can account now for the cost of the lost information $\sum_{i,j=1}^{M} \alpha_{i,j} l_{i,j} x_{i,j}[k]$ in the total reward calculation.

The requirements for maximizing the speed of information processing can be modeled by introducing the reward time decay rate τ in the definition of the gain from information processing. We identify a gain at a time step $[k]$ to be equal to $\tau^k \sum_{i=1}^{M} g_i p_i[k]$. The objective function becomes to maximize the reward equal to:

$$\sum_{k \ge 0} \tau^k \left[\underbrace{\sum_{i=1}^{M} g_i p_i[k]}_{gain} - \underbrace{\sum_{i,j=1}^{M} \alpha_{i,j} l_{i,j} x_{i,j}[k]}_{loss} \right].$$

The following constraints for time step $[k]$ are considered:

- $r_i[k] \le r_i^{MAX}$
- $p_i[k] \le p_i^{MAX}$
- $x_{i,j}[k] \le c_{i,j}^{MAX}$

The dynamics of the agent system can be therefore described as follows:

- received info:

$r_i[0] = r_i^0$, and

$$r_i[k+1] = \left(1 - \pi_i[k] - \sum_{j=1}^{M} \pi_{i,j}[k]\right) r_i[k] + \sum_{m=1}^{M} (1 - l_{m,i}) x_{m,i}[k]$$

- info in "received" buffer

- processed info:

$p_i[0] = 0; p_i[k+1] = \pi_i[k] r_i[k]$

- sent info:

$x_{i,j}[0] = 0; x_{i,j}[k+1] = \pi_{i,j}[k] r_i[k]$

Therefore, the variables $\pi_i[k] \in [0,1]$, $\pi_{i,j}[k] \in [0,1]$ identify the solution to this problem, and the following constraints are applied:

- $\pi_i[k] + \sum_{j=1}^{M} \pi_{i,j}[k] \le 1$
- $r_i[k+1] \le r_i^{MAX} \Rightarrow$

$$\left(1 - \pi_i[k] - \sum_{j=1}^{M} \pi_{i,j}[k]\right) r_i[k] + \sum_{m=1}^{M} (1 - l_{m,i}) \pi_{m,i}[k-1] r_m[k-1] \le r_i^{MAX}$$

- $p_i[k+1] \le p_i^{MAX} \Rightarrow \pi_i[k] \le \dfrac{p_i^{MAX}}{r_i[k]}$
- $x_{i,j}[k+1] \le c_{i,j}^{MAX} \Rightarrow \pi_{i,j}[k] \le \dfrac{c_{i,j}^{MAX}}{r_i[k]}$

B. Single agent information processing

Let's assume that the agent A_i uses all its received information r for internal processing. Accounting for the constraint p_i^{MAX} on maximum volume of information processed at any time step, we conclude that the total reward of such a strategy is equal to

$$g_i(p_i^{MAX} + \tau \cdot p_i^{MAX} + \ldots + \tau^{y-1} \cdot p_i^{MAX} + \tau^y \cdot z) = g_i p_i^{MAX} \frac{1-\tau^y}{1-\tau} + g_i \tau^y \cdot z ,$$

where $y = \left\lfloor \dfrac{r}{p_i^{MAX}} \right\rfloor, z = r - p_i^{MAX} \left\lfloor \dfrac{r}{p_i^{MAX}} \right\rfloor \in (0, p_i^{MAX})$

Then, the reward at A_i is a piece-wise linear function:

$$R_i(r) = g_i p_i^{MAX} \frac{1 - \tau^{\left\lfloor \frac{r}{p_i^{MAX}} \right\rfloor}}{1-\tau} + g_i \tau^{\left\lfloor \frac{r}{p_i^{MAX}} \right\rfloor} \cdot \left(r - p_i^{MAX} \left\lfloor \frac{r}{p_i^{MAX}} \right\rfloor \right) .$$

Function $R_i(r)$ is convex and continuous. Its subgradient is defined as $\nabla R_i(r) = g_i \tau^{\left\lfloor \frac{r}{p_i^{MAX}} \right\rfloor}$.

For more information on strategy selection schemes, see Appendix B.

4. Coupled information-command networks problem

In this section we describe the joint problem of information/command processing, and show how it relates to the simplified information problem described in Section 3.

The difference between information and command types of communication is in the specification of recipients of communication, and parameters/requirements for processing. The information may be rejected, resent, or processed in a similar manner as command. The command may often be combined with information. The difference therefore lies in the restrictions/constraints placed on the transfer of command and information, and the rewards/penalties associated with breaking those

restrictions (for example, penalty for not processing command, penalty for lost information, reward for information processing, etc.).

The volume of information from events is transferred through the network. The information is then converted into command, which is then transferred to agents for processing. We assume that volume of flow does not change when information-to-command transfer occurs. Therefore, we can assume "information" and "command" to be a flow of single type, differentiating only the substructures of the network through which the information and command are routed.

The information and command are assumed to be separable, and can be routed via different paths in the network. This defines a *flow* in the organization.

4.1. Agent responsibilities/tasks

In a coupled information/command problem, agents' roles are expanded (as compared to simplified information network problem, see Subsection 3.1.C), and they accomplish the following tasks:

- *monitor* the events, and receive their information volume;
- *receive* the *information* from other agents;
- *receive* the *command* from other agents;
- *fuse* the received information;
- *fuse* the received command;
- conduct *decision-making* to separate the received *information* into the information to be converted into command and the information flow to be sent to other agents; this can be considered as *information decomposition task*;
- *convert* the information into command;
- conduct *decision-making* to separate the received *command* into the tasks to be processed by the agent ("consumed command") and the command to be sent to other agents; this can be considered as *command decomposition task*;
- *process* the command tasks that are consumed by the agent;
- *send* the information to the rest of the team;
- *send* the command to the rest of the team.

Hence, agent receives three types of flow: (a) information from personal observations; (b) information sent by other agents; and (c) command sent from other agents. As a result, we can view an agent as having the following responsibilities:

(i) monitoring (observe external events);

(ii) decision-making (internal info-to-command conversion);

(iii) processing (execute tasks from command);

(iv) communication (receive/send information and command).

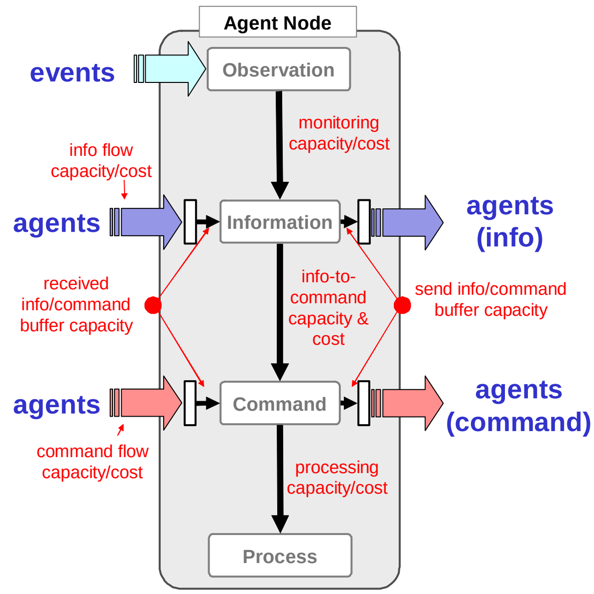

Figure 4.1. Agent-flow model

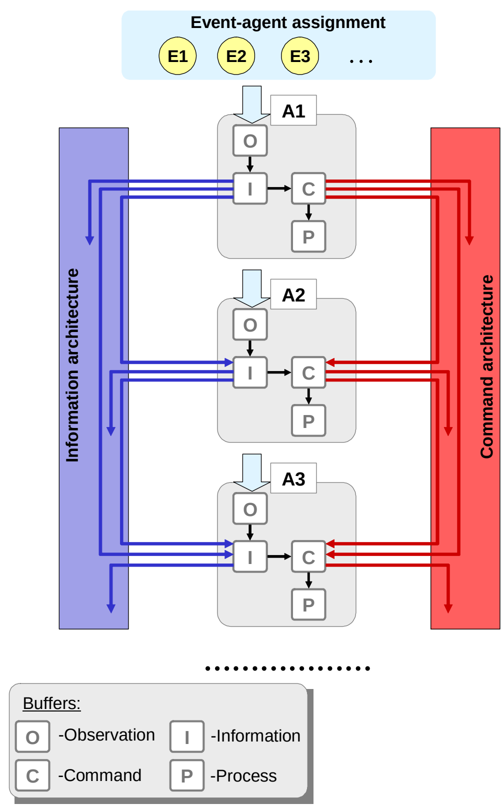

Figure 4.2. Network flow model

We introduce "information" and "command" buffers to differentiate the flow received and sent by an agent. According to the above, the agent flow model is described

in Fig. 4.1 and a model of the flow in the joint network is outlined in Fig. 4.2.

We can see that the information/command model differs from one-network problem formulation of information processing problem in the addition of command flow and command subnode/buffer at each agent. Information and command flows are decomposed into independent networks, which can be considered as networks in two dimensions (see Fig. 4.3). These networks are connected through one-directional information-to-command transfer: flow can leave information network and enter command network, but not other way around. This is accomplished when the flow is routed from "information" buffer at agent node to "command" buffer. Note that such transition is impossible outside agent node shells, and has to occur since there is no flow consumption in the information network.

The information/command problem is therefore similar to single network problem, since the flow type in the network can be assumed non-changing, and the problem has link, node cost and capacity constraints similar to information processing problem. Therefore, all solution approaches described in Appendix A apply for this problem as well. The difference would only be in the treatment of agent nodes, which are replaced in coupled network formulation by their subnodes (information and command buffers).

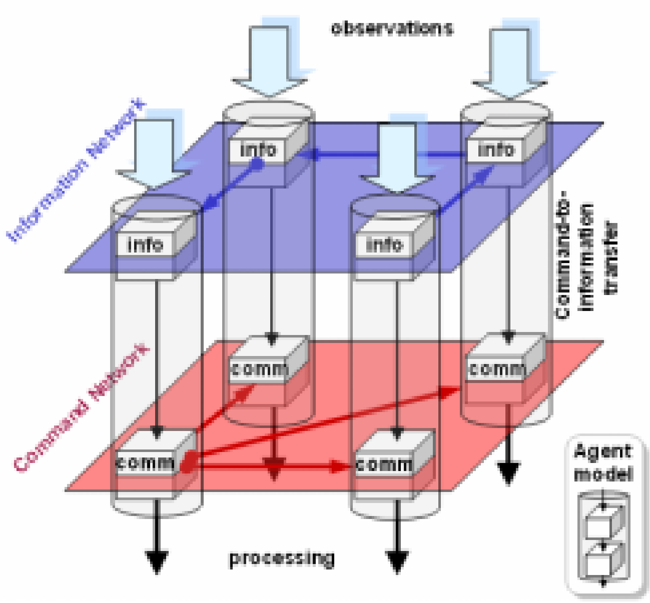

Figure 4.3. Spatial view of information/command model

In the next subsections we outline agent parameters for coupled information/command flow problem.

4.2. Agent parameters

According to agent responsibilities outlined in previous subsection, we define the following parameters for each agent A_i :

Monitoring:

- *Monitoring capacity* $m_i^{\max}$; an agent can monitor and receive information from events of up to $m_i^{\max}$ units;
- *Monitoring efficiency* – the gain obtained by executing a unit of command at the agent: g_i^M ;

Communication:

- *Received information capacity* $r_i^{I,\max}$; an agent can receive total up to $r_i^{I,\max}$ units of information;
- *Received command capacity* $r_i^{C,\max}$; an agent can receive total up to $r_i^{C,\max}$ units of command;
- *Sent information capacity* $s_i^{I,\max}$; an agent can send total up to $s_i^{I,\max}$ units of information;
- *Sent command capacity* $s_i^{C,\max}$; an agent can send total up to $s_i^{C,\max}$ units of command;

Processing:

- *Information processing capacity* $p_i^{I,\max}$; an agent can convert of up to $p_i^{I,\max}$ units of information into command;
- *Command processing capacity* $p_i^{C,\max}$; an agent can execute of up to $p_i^{C,\max}$ units of command;
- *Info processing efficiency* – the gain obtained by converting a unit of information into command at the agent: g_i^I ;
- *Command processing efficiency* – the gain obtained by executing a unit of command at the agent: g_i^C ;

Assignment:

- Event-to-agent assignment: $u_{n,i} \in \{0,1\}$, where $\sum_{i=1}^{M} u_{n,i} = 1$; these parameters define the observers or sources of the flow in the organization.

We define unique capacities for monitoring, information processing, and command processing capacities since agents have limited capabilities and must allocate them to monitoring, decision-making, and processing. Due to limited total workload, the sum of capacities of monitored information, information converted to command, and processed command would be limited.

4.3. Organizational Network

Agents receive and send information and command in the network, which is a collection of links/channels that connect the organizational agents' buffers and allow them to exchange the information. As can be seen from Fig. 4.2, the network consists of two types of links – information links, and command links. Therefore, we can define an agent network in terms of information and communication link capacities $c_{i,j}^I$ and $c_{i,j}^C$, with $\sum_{i,j=1}^{M} c_{i,j}^I + \sum_{i,j=1}^{M} c_{i,j}^C \le C$. Link capacities $c_{i,j}^I$ and $c_{i,j}^C$ determine the maximal volume of correspondingly information and command that can be transferred from agent A_i to agent A_j.

4.4. Problem constraints and objectives

Workload constraints: The monitoring, information processing, and command processing capacities defined above usually refer to the trade-off that an agent exercises in dedicating its time to internal information (information and command processing capacities) and external information (monitoring capacity). Therefore, $m_i^{\max} + p_i^{I,\max} + p_i^{C,\max} = \Omega$, where Ω identifies agent's total workload capacity (we assume for simplicity that agents have identical workload capacities).

Monitoring constraints: Defining the flow received by an agent A_i from events as $m_i; i = 1,...,M$, we have that $m_i \le m_i^{\max}$.

Received information constraints: Defining the information received by an agent A_i as $r_i^I; i = 1,...,M$, we have that $r_i^I \le r_i^{I,\max}$.

Received command constraints: Defining the command received by an agent A_i as $r_i^C; i = 1,...,M$, we have that $r_i^C \le r_i^{C,\max}$.

Sent information constraints: Defining the information sent by an agent A_i as $s_i^I; i = 1,...,M$, we have that $s_i^I \le s_i^{I,\max}$.

Sent command constraints: Defining the command sent by an agent A_i as $s_i^C; i = 1,...,M$, we have that $s_i^C \le s_i^{C,\max}$.

Information processing constraints: Defining the information processed by agent A_i as $p_i^I; i = 1,...,M$, we have that $p_i^I \le p_i^{I,\max}$.

Command processing constraints: Defining the command processed/executed by agent A_i as $p_i^C; i = 1,...,M$, we have that $p_i^C \le p_i^{C,\max}$.

Information link constraints: Defining the information flow along the link from A_i to A_j as $x_{i,j}^I; i, j = 1,...,M$, we have that $x_{i,j}^I \le c_{i,j}^I$.

Command link constraints: Defining the command flow along the link from A_i to A_j as $x_{i,j}^C; i, j = 1,...,M$, we have that $x_{i,j}^C \le c_{i,j}^C$.

Link capacity constraints: The agents in the organization have limited communication capabilities. Therefore, there might be additional constraints imposed on the flow in the agent network. Defining the maximum and minimum capacity along the information and command links from A_i to A_j as $c_{i,j}^{I,\max}, c_{i,j}^{I,\min}, c_{i,j}^{C,\max}, c_{i,j}^{C,\min}; i, j = 1,...,M$, we have that $c_{i,j}^{I,\min} \le c_{i,j}^I \le c_{i,j}^{I,\max}$, $c_{i,j}^{C,\min} \le c_{i,j}^C \le c_{i,j}^{C,\max}$.

Objectives: The design problem is to determine the link capacities in the communication network and routing of the information and command. To identify the cost of the network, we define $\alpha_{i,j}^I$ and $\alpha_{i,j}^C$ - correspondingly the costs per unit of capacity on the information and command link, with the total cost equal to $\sum_{i,j=1}^{M} \left(\alpha_{i,j}^I c_{i,j}^I + \alpha_{i,j}^C c_{i,j}^C \right)$. Various objective functions might be considered: minimization of total network cost, maximization of reward from information and command processing, minimization of information delay, etc. These problems can be addressed by various modeling techniques described in previous sections. Thus, the main factors entering the objective function may include the following:

- *Observation gain* – the gain achieved by information monitoring: $\sum_{i=1}^{M} g_i^M m_i$
- *Information gain* – the gain achieved by processing the information and converting it into command: $\sum_{i=1}^{M} g_i^I p_i^I$
- *Command gain* – the gain achieved by processing the command and its tasks: $\sum_{i=1}^{M} g_i^C p_i^C$

- *Information transfer cost* – the cost of transferring the information flow in information network of organization: $\sum_{i,j=1}^{M} \alpha_{i,j}^{I} x_{i,j}^{I}$
- *Command transfer cost* – the cost of transferring the command flow in command network of organization: $\sum_{i,j=1}^{M} \alpha_{i,j}^{C} x_{i,j}^{C}$

4.5. Defining agent parameters

Agent parameters are defined similarly to information network problem. However, agent's processing gain is different for information- and command-type flow problems. The gain that an agent incurs for information processing (equivalent to information-to-command conversion) is defined, as in Subsection 3.3, using a variant of normalized entropy for agent expertise, which is scaled with mean expertise:

$$g_i^I = \frac{1}{\log \Phi}\left[\sum_{k=1}^{\Phi} \omega_k^i \log \sum_{k=1}^{\Phi} \omega_k^i - \sum_{k=1}^{\Phi} \omega_k^i \log \omega_k^i\right].$$

Command processing describes the consumption of command information, represented in the form of mission and/or management tasks, for processing these tasks. Therefore, the gain from processing the command and the concomitant tasks relates to the efficiency of the agent to handle the information in a higher depth. In this paper, we define the agent gain for processing command of type k equal to its *knowledge depth* (specialization capabilities) in this type ω_k^i. Hence, the gain of processing event that carries information of types from the set $\mathcal{J}$, where $\mathcal{J} \subset \{1,...,\Phi\}$, is equal to $g_i^C = \sum_{k \in \mathcal{J}} \omega_k^i$. To simplify our results, we assume that incoming events are of the same type. The case of events of different types will transform our problem into *multi-commodity flow problem*.

Here, $g_i^C \in [0, \Delta]$, with maximum reached for maximally specialized agents: $\omega_k^i = \Delta, \omega_j^i = 0; j \neq k$. For same-talent Δ agents, the gain is minimal in any information type for maximally generalized agents (that is, when $\omega_k^i = \frac{\Delta}{\Phi}; \forall k = 1,...,\Phi$).

The monitoring gain g_i^M incurred from observing a unit of event information can be defined similarly to command processing gain.

We also define the cost of transferring the unit of information or command from agent A_i to A_j as in Subsection 3.3:

$$\alpha_{i,j}^I = \sum_{k=1}^{\Phi} \mathsf{F}^I(\omega_k^i - \omega_k^j), \text{ and } \alpha_{i,j}^C = \sum_{k=1}^{\Phi} \mathsf{F}^C(\omega_k^i - \omega_k^j).$$

5. Example of methodology application

In this section, we present an example of analysis of designing optimal organizational structure and sensitivity to gain and cost parameters.

5.1. Parameters

Number of information types: We consider 5 information types in this example: $\Phi = 5$.

Mission: We consider a mission consisting of 12 identical events having volume of 1 each. Every event is assumed to carry two information types: $\mathcal{J} = \{1,2\}$.

Table I: Example of cost matrix: information and command link costs ($\times \mu$)

	1	2	3	4	5	6	7	8	9
1	0	2	2	2	2	**2**	**3**	**3**	**2**
2	2	0	0	8	**8**	8	**4**	**4**	4
3	2	0	0	8	**8**	8	**4**	**4**	4
4	2	8	8	0	**1**	**1**	**8**	8	**4**
5	**4**	10	10	1	0	0	10	10	9
6	**4**	10	10	1	0	0	10	10	9
7	5	2	**2**	10	10	10	0	0	2
8	5	2	**2**	10	10	10	0	0	2
9	2	4	4	4	5	5	4	4	0

Organization: Due to the above, the command processing gain of agent A_i is equal to $g_i^C = \omega_1^i + \omega_2^i$. We used the organization consisting of 9 agents, with observation cost = 0, and their capacities, expertise vectors and concomitant gain parameters outlined in Table II. In the following simulations, we used the function $\mathsf{F}^I(x) = \mathsf{F}^C(x) = \mu \max\{x^2, 0\}$ to determine the information and command transfer cost, where μ is a scaling factor (we will explore the sensitivity of μ in the next subsection). Hence, command and information costs (see Table I) are identical and equal to

$$\alpha_{i,j}^I = \alpha_{i,j}^C = \mu \sum_{k=1}^{\Phi} \max\{(\omega_k^i - \omega_k^j)^2, 0\}.$$

5.2. Sensitivity to communication costs

We explore the sensitivity of link cost by varying the scaling factor μ. The increase of μ indicates the increase in the communication cost, with large values of μ restricting communication to 0-cost links. The mission gain (from processing information and command) decreases with increase of μ (see Fig. 5.1).

Table II: Example of agent parameters

Agents	Capacity			Expertise					Observation Gain	Information Gain	Command Gain
	Observation	Information	Command	1	2	3	4	5			
1	0	7	3	1	1	1	1	1	0	5	2
2	0	6	4	0	2	1	2	0	0	3.28	2
3	0	6	4	0	2	1	2	0	0	3.28	2
4	0	6	4	2	0	1	0	2	0	3.28	2
5	4	2	4	1	0	1	0	3	0	2.95	1
6	4	3	3	1	0	1	0	3	0	2.95	1
7	4	3	3	0	3	2	0	0	0	2.09	3
8	4	6	0	0	3	2	0	0	0	2.09	3
9	4	0	6	2	2	1	0	0	0	3.27	4

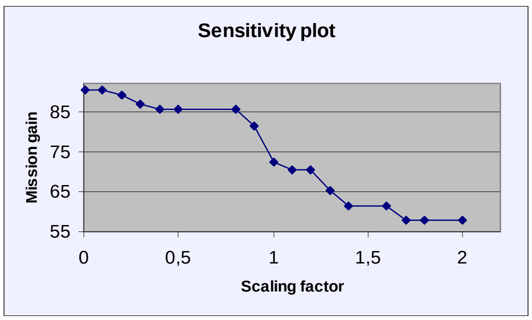

Figure 5.1. Effects of communication cost on mission gain

5.3. Example of design output

We illustrate the design output on the example with $\mu = 0.9$. First, we obtain the optimal information flow strategy (Fig. 5.2) and agent processing assignments (Table III).

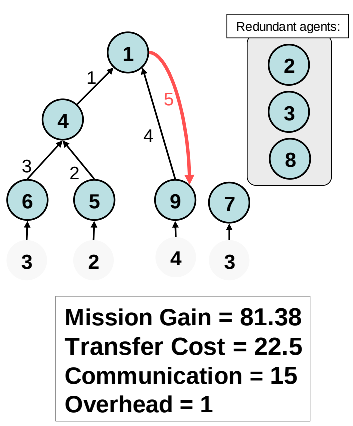

Figure 5.2. Optimal strategy

We notice that 3 agents are redundant in this case. As the result, the optimal organizational structure for this example is a hierarchy, with a single command directed from agent A_1 to A_9. Also, the agent A_7 has an independent strategy, observing, converting to command, and executing its own information without communicating with other agents.

Table III: Example of agents' mission processing parameters

Agents	Processing		
	Observation	Information	Command
1	0	5	0
4	0	4	4
5	2	0	0
6	3	0	0
7	3	3	3
9	4	0	5

6. Example of sensitivity analysis

In this section, we utilize dynamic organizational strategy modeling to illustrate the key concepts of proposed design approach.

6.1. Performance measures

To compare the performance of organizations, we will use the following three metrics:

- *Mission gain* – equal to the total gain from processing information and command: $\sum_{i=1}^{M}\left(g_i^I p_i^I + g_i^C p_i^C\right)$.
- *Inter-agent communication* – equal to the volume of flow among organizational nodes in both information and command networks: $\sum_{i,j=1}^{M}\left(x_{i,j}^I + x_{i,j}^C\right)$.
- *Communication overhead* – equal to the amount of overhead in both information and command networks. The overhead at the agent node is defined as the portion of the flow the agent

receives from other agents that it retransmits in the network without use. For a single agent A_i, this is equal to $r_i^I + r_i^C - \max\{p_i^I - m_i, 0\} - \max\{p_i^C - p_i^I, 0\}$.

6.2. Example of organization

In this sub-section, we present an example of the 9-node organization with hierarchical command network and hybrid information network (both symmetric; see Fig. 6.1), and conduct sensitivity analysis to illustrate the effect of various parameters on organizational performance.

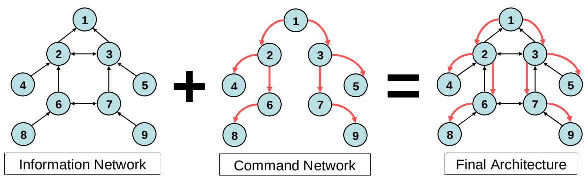

Figure 6.1. Example of organization's topology

We start with the definition of hypothetical agents' efficiency parameters. In this example, we use the characteristics of typical hierarchical organizations, in which the positions of the agents in the hierarchy are determined by agents' expertise in decision-making and task processing. As such, the agents in the top levels of this organization must have higher efficiency in decision-making (due to high generalization capabilities) but lower efficiency in task processing. The agents in the bottom level have higher task processing efficiency due to their specialization. The efficiency/gain parameters for our example are illustrated in Fig. 6.2.

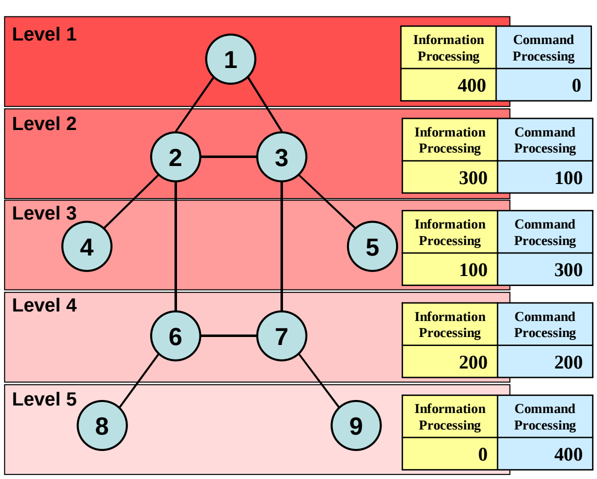

Figure 6.2. Example of agents' efficiency parameters g_i^I and g_i^C

The assignment of agent processing capacities, and information and command network link costs/capacities depend on the agent workload capabilities and network constraints. In the following, we evaluate the sensitivity of organizations to these variables. We start with basis network (with architecture described in Fig. 6.1 and agent efficiency parameters defined in Fig. 6.2), having flow costs of 0 and unlimited link and agent capacities, and proceed by changing one of the variables and evaluating the effects of the change on organizational performance. We consider a simplified example of information and command networks with identical capacities and costs, and identical agent processing capacities. The mission consists of 6 events with volume 2 each. An example of agent observation (event-to-agent allocation) is depicted in Fig. 6.3.

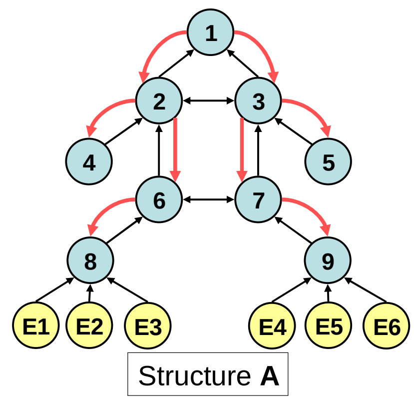

Figure 6.3. Example of event-to-agent allocation

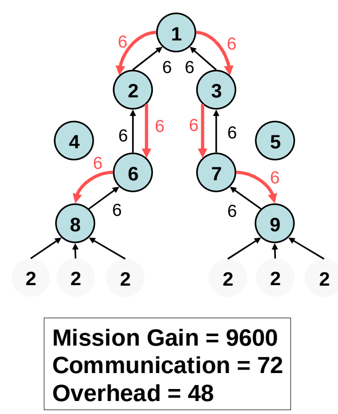

Figure 6.4. Information and command routing in basis organization

We notice that when the link costs are zero, link capacities are unlimited, and agents' processing capacities are unlimited, the organization performs best when all information is processed at the node(s) with maximal information processing efficiency, and the command is processed at the node(s) with maximal command processing efficiency. Also, the

agent monitoring (event-to-agent allocation) does not affect the execution gain. This is due to the fact that in this "basis" organization the flow of information and command is unconstrained and does not incur any cost. In our case, the root agent (A_1) has the maximal information processing gain, while the agents A_8 and A_9 have the maximal command processing gains. Therefore, the information flow is routed to A_1, converted into command at this node, and routed to A_8 and/or A_9 for processing (see Fig. 6.4). Obviously, when link cost/capacity and agent capacity constraints are introduced, the strategy of Fig. 6.4 becomes non-optimal and/or infeasible.

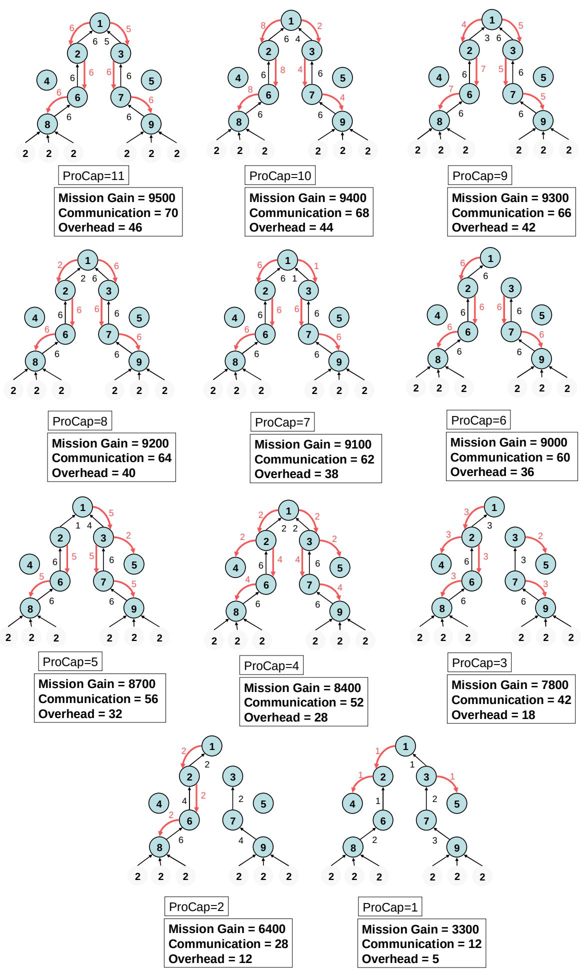

Figure 6.5. Evolution of flow strategy with decrease in agent processing capacity

A. Sensitivity to agent processing capacities

First, we explore the behavior of optimal routing strategy of the organization when the agents' information and command processing capacities are decreased from ∞ to 0. The flow routing in the organization decreases with agent capacities, with a stove-pipe flow of original basis organization changed to more diversified routing involving all agents at lower agent capacities (see Fig. 6.5). This is due to the fact that the total event flow can no longer be processed by single agent, and must be shared among organizational nodes for processing. The mission gain, inter-agent communication, and overhead communication decrease with decrease in agent capacity (see Fig. 6.6), exhibiting sharp decrease with non-linear behavior in $[1,5]$ range. These metrics are constant when agent processing capacities are in $[12,\infty]$ range since all event flow can be processed at a single node.

These results show how the organization is affected by workload constraints which limit the amount of decision-making and processing an agent can perform. In the organizations with significant disparity between efficiency of agent nodes to process information and command (organizations exhibiting high heterogeneity), the mission gain can be improved only if processing capacity of most efficient agents is increased. The gain increase is then achieved by routing the information/command to those agents for processing.

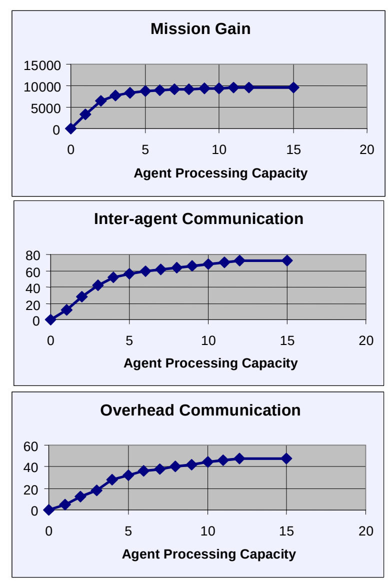

Figure 6.6. Effects of agent processing capacity

B. Sensitivity to link capacities

The basis organization's most efficient strategy (Fig. 6.4) becomes infeasible when agents' processing capacities decrease. The infeasibility lies not in the flow routing in the information and command networks, but in the consumption of the flows by agent nodes. Other parameters that affect the strategy

are the capacities of the links in the information and command networks. These parameters can affect the feasibility of most efficient flow routing. We explore the behavior of optimal routing strategy of the organization when capacities of links in the information and command networks are decreased from ∞ to 0. The communication in the organization decreases, with a stove-pipe flow of original basis organization diminishing and eventually disappearing (see Fig. 6.7). The mission gain, inter-agent communication, and overhead communication decrease with decrease in link capacities (see Fig. 6.8), exhibiting linear behavior. These parameters are constant in $[6, \infty]$ range since all event flows can be routed in the maximally efficient manner.

These results show how the organization is affected by network capacity constraints which limit the feasibility of communication and flow routing in the organization. In the organizations with significant disparity between efficiency of agent nodes to process information and command (organizations exhibiting high heterogeneity), the mission gain can be improved only if network constraints allow to conduct efficient communication among agents.

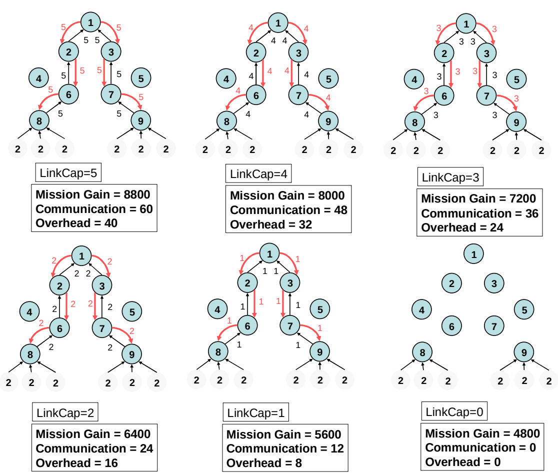

Figure 6.7. Evolution of flow strategy with decrease in link capacity

C. Sensitivity to link costs

The cost parameters do not constrain the feasibility of flow routing. Instead, they affect the trade-off between maximizing the gain from information and command processing and minimizing the cost of communication required to support such processing. Here, we explore the effects of link costs on performance by uniformly increasing the cost on links in the command and information networks from 0 to ∞. As a result, the communication in the organization decreases, with a stove-pipe flow of original basis organization staying constant for long ranges of cost, sharply decreasing for certain cost values, and eventually disappearing (see Fig. 6.9). This is due to the fact that for the specific cost parameter values the old strategy becomes non-optimal, and the new strategy varies greatly from the old one. The mission gain, inter-agent communication, and overhead communication decrease with decrease in link cost (see Fig. 6.10), exhibiting constant behavior in large cost ranges, and steep declines for several "critical" values of link cost.

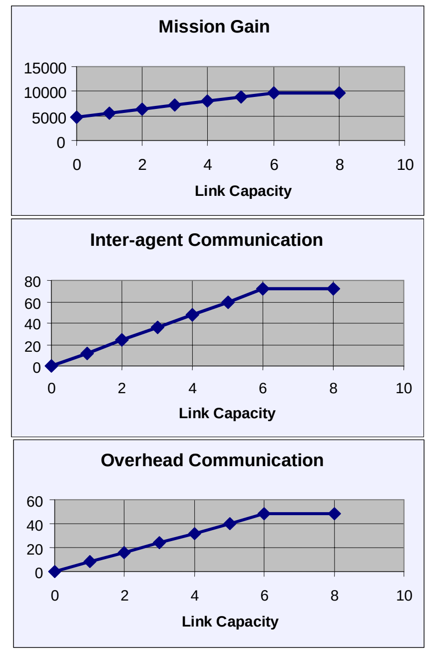

Figure 6.8. Effects of network capacity constraints

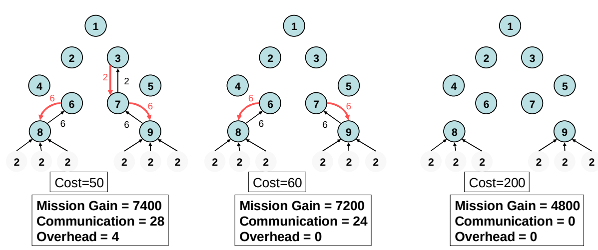

Figure 6.9. Evolution of flow strategy with increase in link cost

These results show how the organization is affected by network cost constraints, which affect the optimal trade-off between flow execution and communication cost. In the organizations with significant disparity between efficiency of agent nodes to process information and command (organizations exhibiting high heterogeneity), the mission gain can be improved only if the cost of communication (flow routing) is decreased. In the organizations with homogeneous agent efficiency and large agent processing capacities,

the flow routing does not significantly affect the overall mission gain. Therefore, the performance of such organizations is not significantly affected by network communication costs.

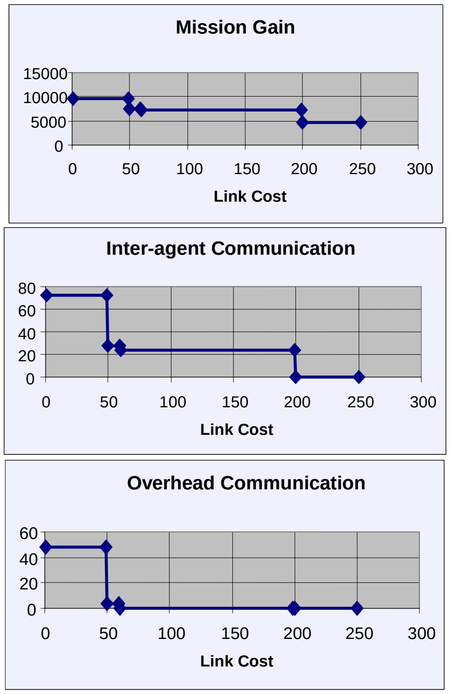

Figure 6.10. Effects of network cost

6.3. Effects of event-to-agent assignment

In the previous subsection, we considered the symmetrical organizational architecture with hierarchical command and hybrid information networks. The examples of the sensitivity analysis compared the performance for a single event-to-agent allocation of Fig. 6.3. This allocation is symmetric, but not optimized for specific organizational constraints.

A. Effects of event allocation

Due to limited talent of agents, their workload, comprised of observation, information processing and command processing, is limited. In the hierarchical organizations this results in the observations (event monitoring) being processed at the lower levels of the hierarchy, while the top agents are dedicated to information processing. In the tree-structured networks such as hierarchies, there are fewer managers than specialists, and the command is easier to distribute among many lower-level agents. Therefore, the average (per manager) amount of processed information (information-to-command conversion) is larger than the average amount of command (task) processing (per specialist). Hence, the task of monitoring is natural to lower-level agents. However, if no such constraints were in place, the optimal event assignment would be to the agents that convert the corresponding event information into command. In the hierarchical organization this would mean assigning all event monitoring to root agent. Clearly, this assignment is unrealistic due to workload constraints.

In the following, we illustrate the effects of event-to-agent assignment on the performance. We compare an assignment of Fig. 6.3 (Structure A in Fig. 6.11) to two alternative event assignments (Structures B & C of Fig. 6.11). Events are assigned to agents with no subordinates. We consider a structure with information and command link capacities equal to 2, unlimited agent processing capacities, and 0 link costs.

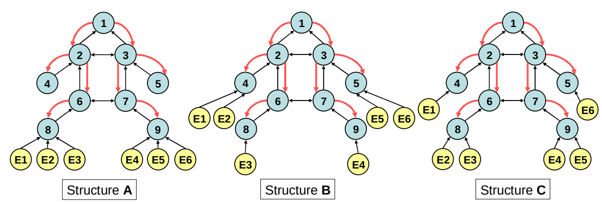

Figure 6.11. Example of event-to-agent assignments

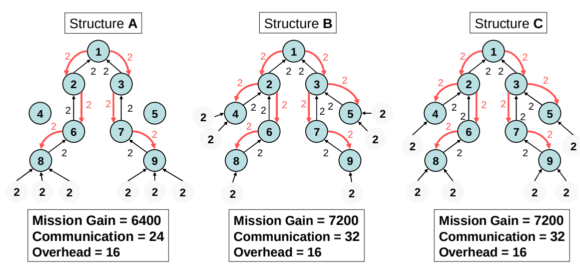

Figure 6.12. Processing strategy for different event assignments

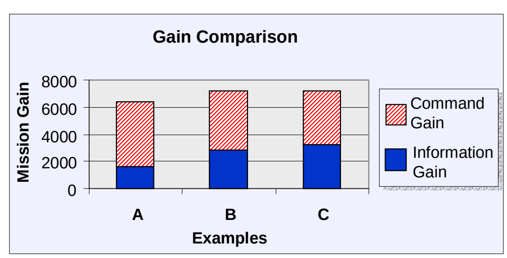

Figure 6.13. Distribution of gains

The resulting strategies are depicted in Fig. 6.12. We can see that alternative event assignments (Structures B & C) produce mission gain improvement of 12.5% compared to old event assignment (Structure A). Structure B allocation produces increase in

information processing gain (although at the expense of some command gain decrease), and the Structure C gives even more increase in information processing gain while command gain is reduced (See Fig. 6.13).

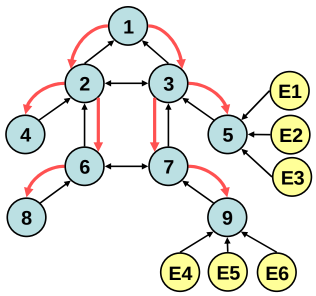

Figure 6.14. Example of asymmetric event-to-agent allocation

B. Asymmetric event allocation

In some situations, the event-to-agent allocation cannot be controlled by the organization. In the case of asymmetric event assignment, the utilization of lateral links in information network allows better distribution of event volume among decision-making agents. Note that so far the lateral links in the information network of a basis organization (Fig. 6.1) have not been utilized. To explore the asymmetric assignment, we consider the basis organization with unlimited information and command link capacities, agents' information and command processing capacities equal to 3, and no link costs. We explore the asymmetric event-to-agent assignment of Fig. 6.14. The resulting optimal strategy is depicted in Fig. 6.15. We can see that lateral links of the information network $3 \rightarrow 2$ and $7 \rightarrow 6$ have been utilized to achieve optimal performance. We conclude that lateral communication capabilities are essential to maintain efficiency in the uncertain environments.

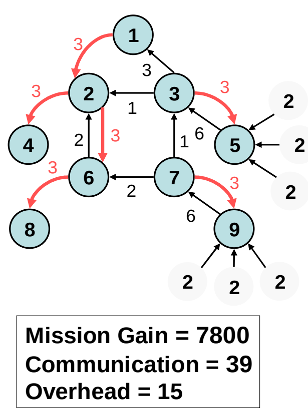

Figure 6.15. Processing strategy for asymmetric event assignments

6.4. Effects of topology

In the above subsections, we illustrated the design analysis on the example of fixed topologies of organizational information and command networks. We have shown how the performance is affected by gain and cost parameters, as well as the event-to-agent allocation. The alternative organizational topologies can also improve performance in the presence of abovementioned constraints. To illustrate this effect, we compare topology of Structure A (Fig. 6.3) with two alternative architectures (Fig. 6.16). We consider the example of fixed information and command networks' link costs = 50, information and command networks' link capacities = 5, and agent information and processing capacities = 4. The resulting strategies are shown in Fig. 6.17. We can see that Structure D produces decrease in flow transfer cost and increase in mission processing gain, and Structure E allows further increase in overall objective value (equal to mission gain less flow transfer cost). The improvement is due to more efficient topologies of the alternative information and command networks, allowing to effectively distribute event volume processing without significant cost of flow transfer. Note that Structure D also allows using fewer agents for mission execution.

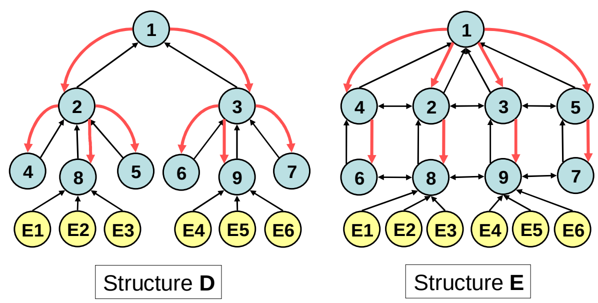

Figure 6.16. Alternative structures

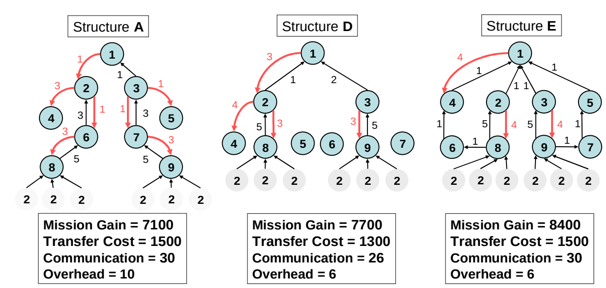

Figure 6.17. Strategies for alternative structures

6.5. Optimal topology

The optimal structure for parameters of previous subsection (information and command networks link costs = 50, information and command networks link

capacities = 5, agent information and processing capacities = 4, and fixed event-to-agent assignment to agents A_8 and A_9) can be obtained by relaxing the topology constraints, and finding the optimal flow routing. The optimal strategy is shown in Fig. 6.18, and one of the structures that can support such strategy is shown in Fig. 6.19.

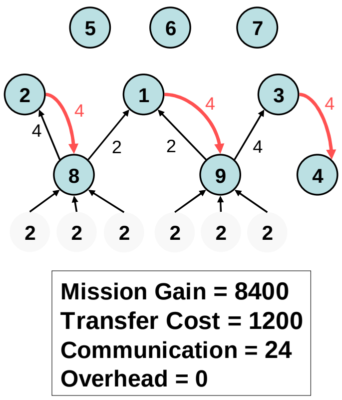

Figure 6.18. Optimal strategy

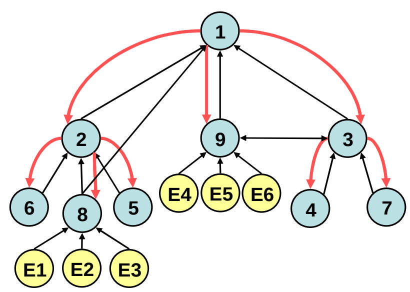

Figure 6.19. Optimal Architecture

7. Conclusions and future research

In this paper, we have explored the decomposition of organizational mission processing into observation, information processing (command generation), command processing (task execution), and communication (transferring information and command). Our methodology consists of modeling the organization as a coupled 2-network structure, where the assignment and routing of information and command processing determines the organizational strategy. The information and command networks (their topologies, communication costs, and capacity constraints) identify the organizational structure. The optimal organizational design problem is then formulated as finding the optimal processing strategy and an organizational structure that can support it. We present our design methodology based on static and dynamic problem formulations. Our methodology also allows comparing different organizational structures and conduct parameter sensitivity and objectives trade-off analysis.

In our research, we have considered a simplified problem formulation, in which the mission consists of identical events, and the event information (as well as the resulting command) can be split and transferred through different paths in the network. Our simplified modeling can be extended to include missions with events of multiple types and information that cannot be split. In our context, this would require to solve the multi-commodity unsplittable flow problem for static problem formulation.

Another extension to our methodology is modeling the network design problem (Bertsekas and Gallager, 1992), (Levchuk *et al.*, 2003) as a capacitated topological design problem to account for average information delay of messages in the organizational networks. We will explore these issues in our future work.

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APPENDIX A: Solution approaches for network model based on static problem formulation

D.1. LP relaxation

By relaxing the integer constraints, we obtain a generic min cost flow problem (see Fig. 3.3), with event nodes having supply constraints, and "process" node having demand constraint.

D.2. Slack variables – 1

We can rewrite a problem as follows:

$$\min \sum_{i,j=1}^{M+1} \alpha_{i,j} x_{i,j}$$

$$s.t. \begin{cases} s_i + \sum_{j=1}^{M+1} x_{j,i} = \sum_{j=1}^{M+1} x_{i,j} + d_i \\ c_{i,j}^{MIN} \le x_{i,j} \le c_{i,j}^{MAX} \\ \sum_{n=1}^{N} e_n u_{n,i} = r_i \\ r_i = s_i \\ \sum_{i=1}^{M+1} u_{n,i} = 1, u_{n,i} \in \{0,1\} \end{cases}$$

By relaxing constraints $r_i = s_i$, we get:

$$L1 = \min \sum_{i=1}^{M+1} \lambda_i s_i + \sum_{i,j=1}^{M+1} \alpha_{i,j} x_{i,j}$$

$$s.t. \begin{cases} s_i + \sum_{j=1}^{M+1} x_{j,i} = \sum_{j=1}^{M+1} x_{i,j} + d_i \\ c_{i,j}^{MIN} \le x_{i,j} \le c_{i,j}^{MAX} \\ \sum_{i=1}^{M+1} s_i = \sum_{n=1}^{N} e_n \end{cases},$$

$$L2 = -\max \sum_{i=1}^{M+1} \sum_{n=1}^{N} \lambda_i e_n u_{n,i}$$

$$s.t. \begin{cases} \sum_{i=1}^{M+1} u_{n,i} = 1 \\ u_{n,i} \in \{0,1\} \end{cases}$$

Hence, $L2 = -\max_{i=1,\dots,M} \lambda_i \sum_{n=1}^{N} e_n$. Therefore, we can find a lower bound to the original problem by solving Lagrangian relaxation. Every step includes solving the

generic min flow cost problem with link costs and flow constraints. Noting that $s_i \le m_i^{MAX}$, the problem would be of the form depicted in Fig. A.1, with one source node replacing all event nodes, and the costs and capacities on the links from this node to every agent node equaling correspondingly λ_i and m_i^{MAX}.

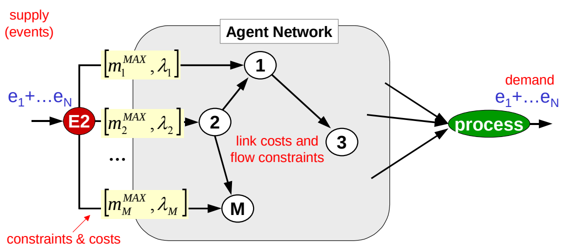

Figure A.1. Relaxed network model

D.2. Slack variables – 2

A flow from event node E_n to agent node A_i is equal to $e_n u_{n,i}$. We can introduce a new variable $y_{n,i} = e_n u_{n,i}$, which satisfies the constraint $y_{n,i} \in \{0, e_n\}$, and $\sum_{i=1}^{M} y_{n,i} = e_n$ ($y_{n,i}$ is equal to the amount of flow monitored by agent A_i from event E_n). We get:

$$\min \sum_{i,j=1}^{M+1} \alpha_{i,j} x_{i,j}$$

$$s.t. \begin{cases} \sum_{n=1}^{N} y_{n,i} + \sum_{j=1}^{M} x_{j,i} = \sum_{j=1}^{M} x_{i,j} + d_i \\ c_{i,j}^{MIN} \le x_{i,j} \le c_{i,j}^{MAX} \\ \sum_{i=1}^{M} y_{n,i} = e_n \\ y_{n,i} = z_{n,i}, z_{n,i} \in \{0, e_n\} \end{cases}$$

Relaxing constraints $y_{n,i} = z_{n,i}$, we obtain:

$$L1 = \min \sum_{i,j=1}^{M} \alpha_{i,j} x_{i,j} + \sum_{n=1}^{N} \sum_{i=1}^{M} \lambda_{n,i} y_{n,i}$$

$$s.t. \begin{cases} \sum_{n=1}^{N} y_{n,i} + \sum_{j=1}^{M} x_{j,i} = \sum_{j=1}^{M} x_{i,j} + d_i \\ c_{i,j}^{MIN} \le x_{i,j} \le c_{i,j}^{MAX} \\ \sum_{i=1}^{M} y_{n,i} = e_n \end{cases}$$ and

$$L2 = -\max \sum_{n=1}^{N} \sum_{i=1}^{M} \lambda_{n,i} z_{n,i}$$

$$s.t. \begin{cases} \sum_{i=1}^{M} z_{n,i} = e_n \\ z_{n,i} \in \{0, e_n\} \end{cases}.$$

Then, $L2 = -\sum_{n=1}^{N} e_n \max_{i=1,\ldots,M} \lambda_{n,i}$. We can see that the relaxed problem requires the iterative calculation of the min cost flow for a network of Fig. 3.3 with added cost of $\lambda_{n,i}$ per unit of flow from event nodes to agent nodes. The iterations are at the costs of these flows. We need to find the updates: $\lambda_{n,i} \leftarrow \lambda_{n,i} + \Delta\lambda_{n,i}$.

APPENDIX B: Strategy selection schemes for network model based on system dynamics problem formulation

A. Single agent single-step reward estimation: Known Parameters

In this subsection, we calculate the decision-making (selection of $\pi_i[k], \pi_{i,j}[k]$ under constraints) at a single node. The objective is to maximize the reward obtained by such a strategy. We assume that all agents' decisions are known except agent A_i, and its decision-making must be optimized.

The information conservation constraints at the agent A_i guide the strategy selection:

$$1 - \frac{r_i^{MAX} - \sum_{m=1}^{M} (1 - l_{m,i}) \pi_{m,i}[k-1] r_m[k-1]}{r_i[k]}$$

$$\le \pi_i[k] + \sum_{j=1}^{M} \pi_{i,j}[k] \le 1$$

$$\pi_i[k] \le \frac{p_i^{MAX}}{r_i[k]}, \; \pi_{i,j}[k] \le \frac{c_{i,j}^{MAX}}{r_i[k]}$$

Also, we must account for constraints in the received buffers of other agents. That is, to have a feasible solution, we must satisfy

$r_j^{MAX} \ge \sum_{m=1}^{M} (1 - l_{m,j}) \pi_{m,j}[k] r_m[k]$. Hence, we must have:

$$\pi_{i,j}[k] \le \frac{r_j^{MAX} - \sum_{\substack{m=1\\ m\neq i}}^{M} (1-l_{m,j})\pi_{m,j}[k]r_m[k]}{r_i[k](1-l_{i,j})}.$$

A gain from variable $\pi_i[k]$ (processing information at the agent A_i) is $g_i p_i[k+1] = g_i r_i[k]\pi_i[k]$.

A loss from variable $\pi_{i,j}[k]$ (sending information from A_i to A_j) is $l_{i,j}x_{i,j}[k+1] = l_{i,j}\pi_{i,j}[k]r_i[k]$.

A gain from variable $\pi_{i,j}[k]$ is equal to the gain that could be obtained by using this information further down the organization network. It is at least the gain that can be obtained at the recipient agent A_j:

$$R(r_j[k] + (1-l_{i,j})\pi_{i,j}[k]r_i[k]) - R(r_j[k]).$$

The problem of optimal one-step flow distribution at agent A_i is formulated in terms of the unknown process rate π_i and flow rates $\{\pi_{i,j}\}$ to maximize the decision gain

$$g_i r_i[k]\pi_i[k] - \sum_{j=1}^{M}\alpha_{i,j}l_{i,j}\pi_{i,j}[k]r_i[k] + \sum_{j=1}^{M} R(r_j[k] + (1-l_{i,j})\pi_{i,j}[k]r_i[k])$$

subject to flow constraints. The problem becomes:

$$\max\, g_i r_i[k]\pi_i[k] - \sum_{j=1}^{M}\alpha_{i,j}l_{i,j}\pi_{i,j}[k]r_i[k] + \sum_{j=1}^{M} R(r_j[k] + (1-l_{i,j})\pi_{i,j}[k]r_i[k])$$

$$s.to \begin{cases} 1 - \dfrac{r_i^{MAX} - \sum_{m=1}^{M}(1-l_{m,i})\pi_{m,i}[k-1]r_m[k-1]}{r_i[k]} \le \pi v_i[k] + \sum_{j=1}^{M}\pi_{i,j}[k] \le 1 \\ 0 \le \pi_i[k] \le \min\left[\dfrac{p_i^{MAX}}{r_i[k]}, 1\right] \\ 0 \le \pi_{i,j}[k] \le \min\left[\dfrac{r_j^{MAX} - \sum_{\substack{m=1\\ m\neq i}}^{M}(1-l_{m,j})\pi_{m,j}[k]r_m[k]}{r_i[k](1-l_{i,j})}, \dfrac{s_{i,j}^{MAX}}{r_i[k]}, 1\right] \end{cases}$$

In this problem formulation, we used the knowledge about decisions of other agents in the constraint definition: $\pi_{m,j}[k], m \neq i$. This can be replaced by estimating the amount sent by other agents using the knowledge available only at A_i.

Let's rewrite the above problem using the following notations:

$$y_1 = \pi_i[k], y_{j+1} = \pi_{i,j}[k]$$

$$\beta_1(y) = g_i r_i[k]y$$

$$\beta_{j+1}(y) = R(r_j[k] + (1-l_{i,j})yr_i[k]) - l_{i,j}\alpha_{i,j}r_i[k]y$$

$$a = 1 - \frac{r_i^{MAX} - \sum_{m=1}^{M}(1-l_{m,i})\pi_{m,i}[k-1]r_m[k-1]}{r_i[k]}$$

$$b_1 = \min\left[\frac{p_i^{MAX}}{r_i[k]}, 1\right]$$

$$b_{j+1} = \min\left[\frac{r_j^{MAX} - \sum_{\substack{m=1\\ m\neq i}}^{M}(1-l_{m,j})\pi_{m,j}[k]r_m[k]}{r_i[k](1-l_{i,j})}, \frac{c_{i,j}^{MAX}}{r_i[k]}, 1\right]$$

Then we have the following problem:

$$\max\ f(y)=\sum_{j=1}^{M+1}\beta_j(y_j)$$

$$s.to\begin{cases} a\le\sum_{j=1}^{M+1} y_j \le 1\\ 0\le y_j\le b_j\end{cases}$$

To have a feasible solution, we need: $\sum_{j=1}^{M+1} b_j \ge a$. This is equivalent to:

$$\min\left[\frac{p_i^{MAX}}{r_i[k]},1\right] + \sum_{j=1}^{M}\min\left[\frac{r_j^{MAX}-\sum_{\substack{m=1\\ m\ne i}}^{M}(1-l_{m,j})\pi_{m,j}[k]r_m[k]}{r_i[k](1-l_{i,j})},\frac{c_{i,j}^{MAX}}{r_i[k]},1\right] \ge 1-\frac{r_i^{MAX}-\sum_{m=1}^{M}(1-l_{m,i})\pi_{m,i}[k-1]r_m[k-1]}{r_i[k]}$$

The problem described above is a non-linear constrained optimization problem with convex objective function, which can be solved using feasible directions method. On each iteration, we find a vector (direction) d such that:

(a) $\nabla f(y)\cdot d>0$ - descent direction;

(b) if $\sum_{j=1}^{M+1} y_j = a$, then $\sum_{j=1}^{M+1} d_j \ge 0$

if $\sum_{j=1}^{M+1} y_j = 1$, then $\sum_{j=1}^{M+1} d_j \le 0$

(c) if $y_j=0$, then $d_j\ge 0$

if $y_j=b_j$, then $d_j\le 0$

Then we find a step size $\alpha\in\left[\frac{a-\sum_{j=1}^{M+1}y_j}{\sum_{j=1}^{M+1}d_j},\frac{1-\sum_{j=1}^{M+1}y_j}{\sum_{j=1}^{M+1}d_j}\right]$, so that $f(y+\alpha\cdot d)$ is minimized, and update a solution:

$$y=y+\alpha\cdot d.$$

One of the approaches is to use a subgradient of objective function $f(y)$ for finding d. First, we find

$$\tilde d_j=\begin{cases}0,\ \text{if}\ \ y_j=0\ \&\ \nabla\beta_j(y_j)<0\\ 0,\ \text{if}\ \ y_j=b_j\ \&\ \nabla\beta_j(y_j)>0\\ \nabla\beta_j(y_j),\ \text{otherwise}\end{cases}$$

Then, we use a greedy search to satisfy constraint (b). Let's assume without loss of generality that values $\tilde d_j$'s are non-decreasing (otherwise, we sort them). Then, if $\sum_{j=1}^{M+1} y_j = a$, we find the smallest k such that $\sum_{j=k}^{M+1}\tilde d_j\ge 0$, and let $d_j=\begin{cases}\tilde d_j,\ \text{if}\ \ j\ge \mathrm{k}\\ 0,\ \text{otherwise}\end{cases}$. If $\sum_{j=1}^{M+1} y_j = 1$, then we find the largest k such that $\sum_{j=1}^{k}\tilde d_j\le 0$, and let $d_j=\begin{cases}\tilde d_j,\ \text{if}\ \ j\le \mathrm{k}\\ 0,\ \text{otherwise}\end{cases}$.

To find the step size, we use a line search to find α such that $f(y+\alpha\cdot d)>f(y)$.

B. Multi-agent single-step reward estimation: Known Parameters

To find a greedy single-step reward optimizing solution for the whole agent network, we need to solve the following problem:

$$\max\sum_{i=1}^{M} g_i r_i[k]\pi_i[k] + \sum_{i=1}^{M}\sum_{j=1}^{M}\left\{R(r_j[k]+(1-l_{i,j})\pi_{i,j}[k]r_i[k])-\alpha_{i,j}\pi_{i,j}[k]l_{i,j}r_i[k]\right\}$$

$$s.to\begin{cases} 1-\frac{r_i^{MAX}-\sum_{m=1}^{M}(1-l_{m,i})\pi_{m,i}[k-1]r_m[k-1]}{r_i[k]} \le \pi_i[k]+\sum_{j=1}^{M}\pi_{i,j}[k]\le 1\\ 0\le\pi_i[k]\le\min\left[\frac{p_i^{MAX}}{r_i[k]},1\right]\\ 0\le\pi_{i,j}[k]\le\min\left[\frac{c_{i,j}^{MAX}}{r_i[k]},1\right]\\ \sum_{m=1}^{M} r_m[k](1-l_{m,j})\pi_{m,j}[k]\le r_j^{MAX}\end{cases}$$

This is a non-linear programming problem with convex piece-wise linear function, $M(M+1)$ variables, and $M(M+3)$ linear constraints.